Biomedical Admissions Test (BMAT)

Section 2: Mathematics
Questions by Topic

M7: Probability

M7: Probability - Questions by Topic

Mark scheme and explanations at the end

1 Margo has a bag containing 15 different sweets. She has 9 toffees, 4 butterscotch sweets and 2 lemon sherberts.

Calculate the probability that she picks out 2 toffees and 1 butterscotch sweet in any order when she removes 3 sweets from the bag.

A $\frac{21}{5}$

B $\frac{864}{2730}$

C $\frac{673}{455}$

D $\frac{2019}{455}$

E $\frac{3}{15}$

2 I have two identical unfair spinners each with 6 segments numbered 1-6. The probability that each spinner gets a 1 is three times as high as the probability of any other outcome, which are all equally likely.

What is the probability that I get a total of 2 when I spin both spinners?

A $\frac{3}{4}$

B $\frac{9}{64}$

C $\frac{3}{8}$

D $\frac{3}{5}$

E $\frac{1}{4}$

3 Jane has two spinners, each with six segments numbered from 1 to 6. One of her spinners is fair but the other spinner is not. The unfair spinner will land on numbers 1 to 5 with equal probability, but lands on number 6 with a different probability. When Jane spins the spinners, the probability that she will get a total of 12 is $\frac{1}{18}$.

What is the probability that Jane gets a total of 3 when she spins the spinners?

A $\frac{2}{45}$
B $\frac{1}{45}$
C $\frac{1}{2025}$
D $\frac{3}{5}$
E $\frac{2}{5}$

4 Last year, a third of the population of Buckinghamshire admitted to having more than 5 glasses of wine a week.

What is the probability that out of a random group of three people who live in Buckinghamshire, exactly one of them will have admitted to drinking more than 5 glasses of wine a week?

A $\frac{4}{9}$
B $\frac{1}{9}$
C $\frac{1}{3}$
D $\frac{4}{27}$
E $\frac{2}{9}$

5 Marco is going to play one game of monopoly and one game of chess.
The probability that he will win the game of monopoly is $\frac{1}{8}$. The probability that he will win the game of chess is $\frac{2}{5}$.

Work out the probability that Marco will win at least one game.

A $\frac{1}{9}$
B $\frac{1}{20}$
C $\frac{19}{40}$
D $\frac{1}{8}$
E $\frac{21}{40}$

6 There are 9 boys and 11 girls in a class. All the students take a geography test. The mean mark for the boys is 44. The mean mark for the girls is 52. Susan was ill on the day of the test. She takes the test a day later and scores 45.

What is the overall mean mark for the whole class?

- **A** $\frac{242}{5}$
- **B** $\frac{1013}{20}$
- **C** $\frac{1013}{21}$
- **D** $\frac{242}{20}$
- **E** $\frac{96}{5}$

7 Simon and Grace are playing air hockey. The probability of Simon scoring a goal and winning a point is p. They are playing by the rules that the first person to score 3 points wins the game, unless the score is 2-1 in which case the first person to get two points ahead of the other wins.

Calculate the probability that Grace wins the match in exactly 4 rounds.

- **A** $3p^3(1-p)^3$
- **B** $p^3(1-p)^3$
- **C** $3p^3(1-p)$
- **D** $3p(1-p)^3$
- **E** $p^3(1-p)$

Solutions

1 **B is the answer**

Each sweet picked is **mutually exclusive** to any other sweet picked because they cannot happen at the same time; picking a toffee cannot happen at the same time as picking a butterscotch because Margo is only picking 1 sweet at a time.

When dealing with the 3 sweet combinations the probabilities are **multiplied,** because the combination is **independent** of all other combinations. This is the AND rule of probability.

For the scenario of Margo picking 3 sweets, there are three possible combinations for her 1st, 2nd and 3rd pick of sweets:

$B + T + T$
$T + B + T$
$T + T + B$

$$\Rightarrow \left(\frac{4}{15} \times \frac{9}{14} \times \frac{8}{13}\right) + \left(\frac{9}{15} \times \frac{4}{14} \times \frac{8}{13}\right) + \left(\frac{9}{15} \times \frac{8}{14} \times \frac{4}{13}\right)$$

Note, this is the same as doing $3 \times \left(\frac{4}{15} \times \frac{9}{14} \times \frac{8}{13}\right)$

$$\Rightarrow P = 3 \times \frac{4 \times 9 \times 8}{15 \times 14 \times 13} \quad \Rightarrow P = 3 \times \frac{288}{2730} \quad \Rightarrow P = \frac{864}{2730}$$

A is incorrect because it has been reached by calculating: $3 \times \frac{9}{15} \times \frac{8}{15} \times \frac{4}{15}$
Remember, the denominator must be decreased by 1 each time a sweet is taken out of the bag because now there is one less sweet in the bag. Furthermore, remember probabilities must be between 0 and 1.
C and D are incorrect. This answer is reached by using the **OR** rule, i.e. P(A or B) = P(A) + P(B). Remember for this question the **AND** rule needs to be used to get the probability that all the events happen (i.e. all 3 sweets are picked).

Exam Tip - When working out probabilities be clear about the AND rule and the OR rule:

The **AND** rule gives P(both events happen) i.e. the probability of both event 1 and event 2 happening, and so you multiply together the two separate probabilities. This is the case when the 2 events are **independent** to each other - the result of one event does not affect the other event.

The **OR** rules gives P(A or B) i.e. the probability EITHER A happens OR B happens. If A and B are mutually exclusive:

P(A or B) = P(A) + P(B)

2 **B is the answer**

The probability of spinning a 2, 3, 4, 5 or 6 are equally likely. Set the probability of spinning each of these values to be x.
The probability of spinning a 1 is three times as likely as spinning a 2, 3, 4, 5 or 6, and therefore the probability of spinning a 1 is $3x$.
Probability always adds up to 1 and so the probability of spinning a 1, 2, 3, 4, 5 or 6 will equal 1:

$$P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1$$
$$\Rightarrow 3x + x + x + x + x + x = 1 \qquad \Rightarrow 8x = 1$$
$$\Rightarrow x = \frac{1}{8}$$

So the probability of spinning a 2, 3, 4, 5 or 6 is $\frac{1}{8}$. The probability of spinning a 1 is three times this and so is $\frac{3}{8}$.

In order to reach a total sum of 2 when both spinners are spun, a 1 must be obtained on each spinner.

$$P(Getting\ 1\ on\ spinner\ 1) \times P(Getting\ 1\ on\ spinner\ 2) = P(Sum\ of\ 2)$$

$$\Rightarrow P(Sum\ of\ 2) = \frac{3}{8} \times \frac{3}{8} = \frac{9}{64}$$

So, the probability when I spin both spinners of getting a total of 2 is $\frac{9}{64}$.

3 **A is the answer**

For Jane to get a total of 12, she must spin a 6 on both spinners. The fair spinner has an equal probability on landing on any of the numbered segments and so for this spinner, the probability of getting a 6 is $\frac{1}{6}$.

The second spinner is not fair as it lands on the segment numbered 6 with a different probability to landing on the segments numbered 1-5. We don't know the probability of getting a 6 on this spinner, so call this probability x.
To calculate the probability of landing on a 6 for this spinner use the **AND rule**:

$$P(Landing\ a\ 6\ on\ spinner\ 1) \times P(Landing\ a\ 6\ on\ spinner\ 2) = P(sum\ of\ 12)$$

$$\Rightarrow \frac{1}{6} \times x = \frac{1}{18} \qquad \Rightarrow x = \frac{1}{18} \div \frac{1}{6} \qquad \Rightarrow x = \frac{1}{3}$$

So, the probability of landing a 6 on spinner 2 is $\frac{1}{3}$. This means the probability of landing on a segment numbered 1 to 5 is $\frac{2}{3}$ overall. Since this spinner lands on numbers 1 to 5 with equal probability, the probability of landing on segment 1, 2, 3, 4 or 5 can be calculated as follows:

$$P(1 \text{ on spinner } 2) = P(2 \text{ on spinner } 2) = P(3 \text{ on spinner } 2) = P(4 \text{ on spinner } 2) = P(5 \text{ on spinner } 2) = y$$

$$\Rightarrow y = \frac{2}{3} \div 5 = \frac{2}{15}$$

The possible ways for Jane to get a 3 when she spins the spinner are to get the following **combinations**:

Fair spinner: 1 Unfair spinner : 2

OR

Fair spinner: 2 Unfair spinner: 1

Since Jane can either get a 1 and a 2 on spinner 1 and spinner 2 respectively, OR a 2 and then a 1 on spinner 1 and spinner 2 respectively, the OR rule is used here. Call the probability she gets a 1 and 2 on spinner 1 and spinner 2 respectively "A" and the probability she gets a 2 and then a 1 on spinner 1 and spinner 2 respectively "B".

$$P(A \text{ or } B) = P(A) + P(B)$$

To determine the probability of event "A" happening:

$$P(A) = P(\text{Landing a } 1 \text{ on spinner } 1) \times P(\text{Landing a } 2 \text{ on spinner } 2)$$
$$\Rightarrow P(A) = \frac{1}{6} \times \frac{2}{15} = \frac{1}{45}$$

To determine the probability of event "B" happening:

$$P(B) = P(\text{Landing a } 2 \text{ on spinner } 1) \times P(\text{Landing a } 1 \text{ on spinner } 2)$$
$$\Rightarrow P(B) = \frac{1}{6} \times \frac{2}{15} = \frac{1}{45}$$

Now that the probabilities for events "A" and "B" happening are known, the **OR rule** can be used:

$$P(A \text{ or } B) = P(A) + P(B) \qquad \Rightarrow P(A \text{ or } B) = P(\text{sum of } 3) = \; = \frac{1}{45} + \frac{1}{45} = \frac{2}{45}$$

And so the probability of spinning a total of 3 is $\frac{2}{45}$.

4 **A is the answer**

Probability of admitting to drinking more than 5 glasses of wine a week = $\frac{1}{3}$
Probability of not admitting to drinking more than 5 glasses of a wine a week = $1 - \frac{1}{3} = \frac{2}{3}$

In three people, all the combinations of 1 person admitting to drinking more than 5 glasses of wine a week are:

Person 1	Person 2	Person 3	Probability
Yes	No	No	$\frac{1}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{4}{27}$
No	Yes	No	$\frac{2}{3} \times \frac{1}{3} \times \frac{2}{3} = \frac{4}{27}$
No	No	Yes	$\frac{2}{3} \times \frac{2}{3} \times \frac{1}{3} = \frac{4}{27}$

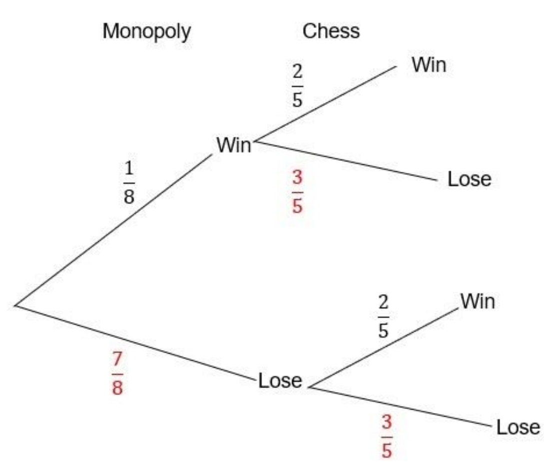
We can use the **OR rule** to obtain the probability of getting one of these combinations when three people are tested at random:

$$P(\text{One person out of three admits to drinking } > 5 \text{ glasses}) = \frac{4}{27} + \frac{4}{27} + \frac{4}{27} = \frac{4}{9}$$

The probability of exactly one person out of three people asked admitting having more than 5 glasses of wine a week is $\frac{4}{9}$.

5 **C is the answer**

For questions involving separate events such as this one (playing a game of monopoly and playing a game of chess), it can be really helpful to draw a tree diagram. In the question, we have been told the probability of Marco winning each game. Using this, we can calculate the probability of him losing at each game:



The question asks for the probability Marco wins **at least** one game. With "At least questions", the formula is $1-(probability\ of\ 'less\ than'\ the\ number\ stated\ in\ the\ at\ least\ part)$

So in this question, the formula to use is:

$$P(At\ least\ one\ win) = 1 - P(Less\ than\ 1\ win).$$

Less than 1 win is 0 wins, which is the same as saying he loses all his games.

$$\Rightarrow P\ (Marco\ wins\ at\ least\ one\ game)\ =\ 1 - P(Marco\ loses\ all\ games)$$

$$P(Marco\ loses\ all\ games)\ = P(Loses\ Monopoly)\ \times P(Loses\ Chess)$$

The **AND rule** is used because we need to calculate the probability both events happen, i.e. Marco loses at monopoly AND Marco loses at chess. The probability Marco loses monopoly is $\frac{7}{8}$. The probability Marco loses chess is $\frac{3}{5}$. So,

$$P(loses\ all\ games)\ = \frac{7}{8} \times \frac{3}{5}\ =\ \frac{21}{40}$$
$$\Rightarrow P(Marco\ wins\ at\ least\ one\ game)\ =\ 1\ -\ \frac{21}{40}$$
$$\Rightarrow P(Marco\ wins\ at\ least\ one\ game)\ =\ \frac{19}{40}$$

6 **C is the answer**

Work out the total score of the boys:

$Sum\ of\ boy's\ score\ =\ 9\ \times 44\ = 396$

Work out the total score of the girls, excluding Susan's score:

$Sum\ of\ girl's\ score\ =\ 52\ \times 11\ =\ 572$

Add Susan's score to get the new total score of the girls:

$Total\ sum\ of\ girl's\ score\ =\ 572\ + 45\ =\ 617$

Calculate the new mean: $mean\ =\ \frac{sum\ of\ all\ values}{total\ no.\ of\ values}$. Remember, the total number of students has increased by one because Susan has to be added to the group of girls.

$$\Rightarrow Mean\ score\ =\ \frac{396+617}{9+12} \qquad \Rightarrow Mean\ score\ =\ \frac{1013}{21}$$

7 **D is the answer**

Let A represent the event of Simon winning the point and let B represent the event of Grace winning the point.

The list of possibilities for Grace winning in 4 rounds is:

$$ABBB,\ BABB,\ BBAB$$

A represents the probability of Simon winning the point, which is p. If the probability of Simon winning the point is p, the probability of Grace winning the point is $1-p$.

Each of the three possible paths for Grace to win the match in 4 rounds has the probability:

$$p \times (1-p) \times (1-p) \times (1-p) \text{ in some order}$$

$$\Rightarrow P(ABBB) = P(BABB) = P(BBAB) = p(1-p)^3$$

There are three possible paths and so the final answer is

$$P(\textit{Grace wins in 4 rounds}) = 3 \times p(1-p)^3$$

$$\Rightarrow P(\textit{Grace winds in 4 rounds}) = 3p(1-p)^3$$